

UFIFT-HEP-98/11
 hep-ph/9807341
 July 1998

The $Z_2 \times Z_2$ Orbifold and the SUSY Flavor Problem ^a

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The characteristic property of the $Z_2 \times Z_2$ orbifold is its cyclic permutation symmetry. It is argued that this property may be instrumental in explaining simultaneously the fermion mass hierarchy and the squark mass degeneracy. Detailed studies in free fermionic models that preserve the cyclic permutation symmetry of anomalous $U(1)$ charges of the three generations are discussed.

1 The SUSY Flavor Problem

The flavor problem in supersymmetric extensions of the Standard Model is especially interesting. On the one hand flavor dependent symmetries are motivated by the need to explain the hierarchical fermion mass spectrum. On the other hand, the absence of Flavor Changing Neutral Currents at an observable rate suggests the need for flavor independent symmetries, which force squark mass degeneracy. Thus, the natural question arises how can these, seemingly orthogonal, symmetries coexist. In supersymmetric, two doublet Higgs models, the first problem requires that the Yukawa couplings which couple the fermions to the Higgs fields exhibit an hierarchical pattern. The second problem requires that the soft SUSY breaking parameters are either highly degenerate, or are aligned with the ordinary quark Yukawa matrices. In the context of supersymmetric field theories, the various parameters are fixed to agree with the data, and acceptable solutions can of course be found. The problem, however, becomes more interesting when one tries to unify the gauge interactions with gravity. Superstring models provide useful laboratories to study such theories. In this case the flavor symmetries, in a given model, are imposed and cannot be chosen arbitrarily. While the study of string models is still in its infancy, it is in general indeed expected that superstring models give rise to non-universal soft SUSY breaking parameters¹. Therefore, string models that can on the one hand give rise to flavor dependent symmetries that are needed to explain the fermion flavor mass pattern while, on the other hand producing a mechanism to produce nearly flavor degenerate soft SUSY breaking param-

^aInvited talk presented at PASCOS 98, Boston MA, March 22–29 1998.

ters, may be of special interest. Furthermore, recent progress in understanding nonperturbative aspects of string compactifications suggests that the desired symmetries should also remain in the nonperturbative extension of potentially realistic superstring models, which are constructed at weak coupling.

2 Realistic Free fermion Models

In this talk I discuss a class of string models that can indeed resolve the fermion mass puzzle on the one hand, while providing a robust symmetry structure that may, at least partially, alleviate the sfermion flavor problem. We have to caution however that a completely mutually satisfactory solution to both problems, in the context of string solutions, can only be obtained once the full fermion spectrum is computed quantitatively and similarly for the sfermions soft masses. However, short of this ambitious, and still distant, goal, we can still observe qualitative properties of specific string compactifications and suggest scenarios how both problems can be resolved in such an eventual calculation.

There are several possible ways to try to construct realistic superstring models. One possibility is to construct models with a GUT gauge group, like $SU(5)$ $SO(10)$ or E_6 , which is broken to the Standard Model gauge group at an intermediate energy scale². Another possibility is to construct superstring models with semi-simple GUTs, like $SU(3)^3$ ³, $SU(5) \times U(1)$ ⁴ or $SO(6) \times SO(4)$ ⁵. Finally, we can construct superstring models in which the non-Abelian factors of the Standard Model are obtained directly at the string level^{6,7,8}. A realistic model of unification must satisfy a large number of restrictive phenomenological constraints, a few of which are listed below.

1. Gauge group $\longrightarrow SU(3) \times SU(2) \times U(1)_Y$
2. Contains three generations
3. Proton stable ($\tau_P > 10^{30+}$ years)
4. Agreement with $\sin^2 \theta_W$ and α_s at M_Z
5. N=1 supersymmetry (or $\tilde{N}=0$)
6. Contains Higgs doublets \oplus potentially realistic Yukawa couplings
7. Light left-handed neutrinos
 8. $SU(2) \times U(1)$ breaking
 9. SUSY breaking
 10. No flavor changing neutral currents
 11. No strong CP violation
 12. Exist family mixing and weak CP violation
13. + ...

The last criteria provides a strong motivation for the study of superstring models. It is often stated that superstring theory contains an enormous number of consistent models and we have no mechanism to select among these vacua. This is especially significant in view of the fact that even the deeper understanding of nonperturbative aspects of string theory, gained in recent years, does not seem to lessen the problem. However, this notion is very misleading. Nearly all of these consistent vacua do not satisfy even the first two necessary requirements. Moreover, many of the models, that have been labeled as “realistic”, satisfy *only* the first two criteria, plus possibly existence of possible light Higgs doublets and couplings between the fermions and Higgs doublets that can be identified as Yukawa couplings. However, as we go down the list nearly all string models that have been constructed to date are excluded. Take, for example, the fourth requirement in the list above, and consider the three generation standard-like orbifold models. Such three generation models were constructed in the Z_3 , Z_7 and $Z_2 \times Z_2$ orbifold. In the first two cases one finds a weak-hypercharge embedding with the normalization $k_Y > 5/3$, whereas the last one produced three generation models with the standard $SO(10)$ embedding, *i.e.* with $k_Y = 5/3$. It is precisely for this reason that the superstring models, which are based on the $Z_2 \times Z_2$ orbifold, can be in agreement with the measured values of $\alpha_s(M_Z)$ and $\sin^2 \theta_W(M_Z)$. It is important to emphasize that this does not mean that one of string models that have constructed to date is necessarily the correct string vacuum. Rather, it shows that the $Z_2 \times Z_2$ orbifold compactification contains the desired ingredients that can explain many of the phenomenological requirements in the above list. Thus motivating the hypothesis that the true string vacua is a $Z_2 \times Z_2$ orbifold, in the vicinity of the free fermionic models.

The superstring models, based on a $Z_2 \times Z_2$ orbifold compactification have been studied in the framework of the free fermionic construction⁹, in which the moduli of the compactified dimensions are frozen at the value, where they can be realized as free fermions propagating on the world-sheet. The models are specified in terms of a set of boundary condition basis vectors for all world-sheet fermions, which are constrained by the string consistency requirements. The physical spectrum is obtained by applying the generalized GSO projections. The low energy effective field theory is obtained by S-matrix elements between external states. The Yukawa couplings and higher order nonrenormalizable terms in the superpotential are obtained by calculating correlators between vertex operators. For a correlator to be non vanishing all the symmetries of the model must be conserved. The boundary condition basis vectors

completely define the spectrum and the symmetries of a given string model, and therefore its phenomenological properties.

The first five basis vectors $\{\mathbf{1}, S, b_1, b_2, b_3\}$, in the models of interest here, consist of the so called NAHE set, which yields after the generalized GSO projections an $N = 1$ supersymmetric $SO(10) \times SO(6)^3 \times E_8$ gauge group. The three sectors b_1 , b_2 and b_3 correspond to the three twisted sectors of the $Z_2 \times Z_2$ orbifold. The correspondence is explicitly demonstrated by adding to the NAHE set the basis vector X with periodic boundary conditions for the right-moving world-sheet fermions $\{\psi^{1,\dots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\}$ ^{10,11}. The gauge group in this case is $E_6 \times U(1)^2 \times SO(4)^3 \times E_8$, with 24 generations in the 27 of E_6 . The same model is constructed in the bosonic language by specifying the background metric and antisymmetric tensor field and then moding out by the $Z_2 \times Z_2$ discrete symmetry. In the realistic models the sign of the GSO phase $c(X; \xi = \mathbf{1} + b_1 + b_2 + b_3)$ is flipped, which breaks $E_6 \rightarrow SO(10) \times U(1)$, and $E_8 \rightarrow SO(16)$. The 27 representations of E_6 now become 24 representations in the chiral 16 of $SO(10)$, plus 24 representations in the vectorial 16 of $SO(16)$. The $U(1)$ current that was previously embedded in E_6 now becomes an anomalous, flavor independent, $U(1)$ symmetry. The two orthogonal $U(1)$ combinations are, flavor dependent, anomaly free $U(1)$ currents.

There are two features that are important to note from the discussion above. The first is the appearance of anomalous $U(1)$ symmetry, as a result of the breaking of $E_6 \rightarrow SO(10) \times U(1)^2$, which is family universal. The second is the cyclic permutation symmetry between the three sectors b_1 , b_2 and b_3 with respect to their left- and right-moving quantum numbers under the world-sheet currents. This cyclic permutation symmetry is the characteristic property of the $Z_2 \times Z_2$ orbifold compactification, with the standard embedding of the gauge connection.

The massless spectrum of the E_6 model contains also three pairs of $27 \oplus \overline{27}$ from the untwisted sector. In the $SO(10) \times U(1)$ model these states are broken to three pairs of the vectorial 10 representation of $SO(10)$. These states also respect the cyclic permutation symmetry of the $Z_2 \times Z_2$ orbifold, and give rise to the Yukawa couplings $16_i 16_i 10_i$, where i denotes the i^{th} twisted sector.

The three generation free fermionic models are constructed by adding three additional boundary condition basis vectors beyond the NAHE set⁸. These break the $SO(10)$ and E_8 gauge groups to one of their subgroups. The flavor $SO(6)$ symmetries are broken to a product of $U(1)$ factors. The twisted sectors give three generations, one from each b_1 , b_2 and b_3 . The vectorial untwisted representations produce three pairs of Higgs representations. The additional vectors, beyond the NAHE set, give rise to additional massless spectrum which varies between models. These representations are in general vector-like. In all

models that have been studied in detail there is one vector combination which produces additional light Higgs representations. These additional Higgs representations play an important role in the analysis of fermion mass matrices. The other states from the sectors beyond the NAHE set are mostly exotic vector-like states with fractional electric charge or fractional $U(1)_{Z'}$. For appropriate choices of flat directions these states can decouple from the massless spectrum and are not of interest here.

In general, the additional boundary conditions basis vectors, beyond the NAHE set break the cyclic permutation symmetry. What is remarkable is that in some models the cyclic permutation symmetry of the three generations with respect to their $U(1)$ quantum numbers^b is preserved. As a result in these models the anomalous $U(1)$ is *family universal*. Interestingly, the boundary condition basis vectors that preserve this cyclic permutation symmetry have some specific properties^{13,15}.

3 Fermion Mass Hierarchy

We can now start to see how, with the above general structure and in the presence of a family universal anomalous $U(1)$ both the fermion and sfermion flavor problem can be resolved simultaneously. The details of the analysis has been presented elsewhere^{12,14}, so only the qualitative structure is discussed here. The fermion mass terms are of the form $cgf_i f_j h \phi^{N-3}$ or $cgf_i f_j \bar{h} \phi^{N-3}$, where c is a calculable coefficient, g is the gauge coupling at the unification scale, f_i, f_j are the fermions from the sectors b_1, b_2 and b_3 , h and \bar{h} are the light Higgs doublets, and ϕ^{N-3} is a string of Standard Model singlets that get a VEV and produce a suppression factor $(\langle\phi\rangle/M)^{N-3}$ relative to the cubic level terms. Each generation from a sector b_j is charged with respect to the left-moving, $U(1)_{L_j}$ and $U(1)_{L_{j+3}}$, and right-moving $U(1)_{R_j}$ and $U(1)_{R_{j+3}}$, symmetries. Each Higgs doublet pair from the NS sector is charged under $U(1)_{R_j}$. Consequently at the cubic level only the couplings $\{u_j Q_j + N_j L_j\} \bar{h}_j$ and $\{d_j Q_j + e_j L_j\} h_j$ are allowed. Note that each generation couples to a different Higgs pair, and that at this level the cyclic permutation symmetry is retained. As the anomalous $U(1)$ Fayet–Iliopoulos term breaks supersymmetry near the Planck scale, we must assign VEVs to some Standard Model singlets, along flat F and D directions. In this process some of the nonrenormalizable terms become effective renormalizable operators¹⁶. At the same time some of the Higgs doublet representations receive large mass. For specific solutions only two Higgs doublets remain massless down to the electroweak scale. We should warn again that the full solution to the Higgs mass spectrum can only

^bbefore the anomalous and anomaly free combinations are taken.

be obtained once we have a full solutions to the fermion mass spectrum. That is VEVs that enter the fermion mass terms also affect the Higgs mass matrix. For our purpose here we can make the assumption that the light Higgs spectrum consist of two Higgs doublets and examine the qualitative pattern of the fermion mass matrices that arises. Analysis of the nonrenormalizable terms up to order $N = 8$ reveals the following structure

$$M_U \sim \begin{pmatrix} \epsilon, a, b \\ \tilde{a}, A, c \\ \tilde{b}, \tilde{c}, \lambda_t \end{pmatrix}; \quad M_D \sim \begin{pmatrix} \epsilon, d, e \\ \tilde{d}, B, f \\ \tilde{e}, \tilde{f}, C \end{pmatrix}; \quad M_E \sim \begin{pmatrix} \epsilon, g, h \\ \tilde{g}, D, i \\ \tilde{h}, \tilde{i}, E \end{pmatrix},$$

where $\epsilon \sim (\Lambda_{Z'}/M)^2$. The diagonal terms in capital letters represent leading terms that are suppressed by singlet VEVs, and $\lambda_t = O(1)$. The mixing terms are generated by hidden sector states from the sectors $b_j + 2\gamma$ and are represented by small letters. They are proportional to $(\langle TT \rangle/M^2)$. The states from the sector b_3 are identified with the lightest generation.

The important new feature which arises from this analysis is that the Higgs spectrum at the string scale is more complicated than a simple two Higgs doublet model. Then although there exist a permutation symmetry at the level of the NAHE set this permutation symmetry interchanges between the twisted sectors, as well as between the untwisted Higgs doublets. The permutation symmetry is partially broken by the additional boundary condition basis vectors, beyond the NAHE set, and is broken further by the choices of flat directions. Consequently, only two of the Higgs representations, say \bar{h}_1 and h_{45} , remain light. Only the top couples to \bar{h}_1 at the cubic level and the mass terms for the lighter quarks and leptons arise from, successively suppressed, higher order terms.

4 Sfermion Mass Degeneracy

Although the permutation symmetry is broken in the fermion mass sector, the charges with respect to the anomalous $U(1)$ are family universal. This is the case, for example, in the model of ref. ⁷. This offers an intriguing possibility. If the dominant source of supersymmetry breaking is the anomalous $U(1)$ then the resulting soft squark masses are family universal. Supersymmetry breaking will occur, at hierarchically small scale if there is a mass term, $m\Phi\bar{\Phi}$, for some Standard Model singlet, which is charged under the anomalous $U(1)$. The effective potential then takes the form

$$V = \frac{g^2}{2} \sum_{\alpha} D_{\alpha}^2 + m^2(|\Phi|^2 + |\bar{\Phi}|^2) \quad (1)$$

where D_α are the various $U(1)$ D -terms, and we assumed a common coupling g at the unification scale, to simplify the analysis. Extremizing the potential it is found that SUSY is broken. Furthermore, for a specific solution of the F and D flatness constraints it is found that the mass term m is hierarchically suppressed and that in the minimum the D -terms of the family universal $U(1)$'s are nonzero, whereas those of the family dependent $U(1)$ vanish. This solution therefore provides an example how the squark mass degeneracy may arise, provided that the dominant component that breaks supersymmetry is the anomalous $U(1)$ D -term. Furthermore, the mass term m , which breaks supersymmetry, can be hierarchically small relative to the Planck scale. This is because such a term must arise from nonrenormalizable terms that contain hidden sector condensates. The condensation scale in the hidden sector is determined by its gauge and matter content. For example, in the model of ref ⁷ we found a cubic level flat $F - D$ solution, with the mass term m induced at order $N = 8$, by matter condensates of the hidden $SU(5)$ gauge group ¹⁴.

The general fermion mass pattern that we found, as well as the flavor universality of the anomalous $U(1)$ are an intrinsic reflection of the underlying $Z_2 \times Z_2$ orbifold structure. Therefore, we see that the $Z_2 \times Z_2$ orbifold contains the intrinsic structure that is needed in order to understand both the fermion mass spectrum as well as the sfermion mass degeneracy. We should remark, however, that we used different flat solutions to analyze each of the scenarios separately. Whether or not it is possible to find a solution that produces the qualitative fermion mass pattern as well as the sfermion mass degeneracy *simultaneously*, still remains to be seen.

SUSY breaking by a family universal anomalous $U(1)$ can produce the desired squark degeneracy. However, the gaugino masses can (at best) only arise at the one loop level in string perturbation theory. In this case the gluino masses are approximately give by $m(\lambda_a) \approx \lambda' \langle F(\bar{\Phi}) \langle \Phi \rangle / M_{Pl}^2 \approx \lambda' \epsilon m$, where $\lambda' \leq \mathcal{O}(1)$ and $\epsilon \approx 1/50$. This induces the hierarchy ¹⁴

$$[m^2(\tilde{q}_i) \approx Q_A^i(\sqrt{3}m^2/2)] > [\Delta m_{\tilde{q}_i}^2 \approx \lambda \epsilon (m^2/2)] > [m_{\lambda_a}^2 \approx \lambda' \epsilon^2 m^2] \quad (2)$$

Because of this hierarchy, it is clear that if SUSY breaking proceeds entirely through anomalous $U(1)$, the gluinos typically would be rather light. From (4), one obtains: $m_{\tilde{g}} \approx 2\lambda' \epsilon m_{\tilde{q}} \approx \lambda'(20-60)\text{GeV}$, for $m_{\tilde{q}} \approx (1-3)\text{TeV}$; this may be too light, compared to the observed limit on $m_{\tilde{g}}$ of 130GeV , unless $\lambda' \geq 2$ and $m_{\tilde{q}} \geq 3\text{TeV}$. To make matters worse, for string solutions, as considered here, λ' vanishes at tree level and can only arise through quantum loops; thus it is expected to be small. This suggests that SUSY breaking through anomalous $U(1)$, quite plausibly, is accompanied by an additional source which provides the dominant contribution to gluino masses ($\sim (1 - \text{few})(100\text{GeV})$), while

preserving the squark-degeneracy, obtained through $U(1)_A$. The interesting possibility is that of SUSY breaking through a combined dilaton-anomalous $U(1)$ scenario. Each of these scenarios separately encounters some difficulties. That of the anomalous $U(1)$ has the problem of small gaugino masses, whereas the dilaton dominated scenario has the problem with color and electric charge breaking. Such a combined analysis may also be instrumental in trying to resolve the dilaton stabilization problem¹⁷.

We have seen that the $Z_2 \times Z_2$ orbifold has an intriguing structure, which is very appealing from the point of view of the fermion and sfermion masses. Several other properties of this class of superstring compactification are worth noting. The first is the fact that the chiral generations from the twisted sectors b_1 , b_2 and b_3 carry positive charges under the anomalous $U(1)$. This is an important property, as otherwise SUSY breaking through the anomalous $U(1)$ would lead to color and electric charge breaking. This positivity of the squarks and sleptons charges under $U(1)_A$ is again a reflection of the $Z_2 \times Z_2$ orbifold structure and the partial embedding of the anomalous $U(1)$ in $E_6 \rightarrow SO(10) \times U(1)_A$. The string models contain also color, and electrically, charged, vector-like states, that may have negative $U(1)_A$ charge. However, in all the cases that we examined these states receive large intermediate mass which over-compensates for the $U(1)_A$ D -term contribution. It is also important to note that the universality structure of the $Z_2 \times Z_2$ orbifold is also reflected in other sectors of the theory. In particular for the untwisted moduli. Thus, even if SUSY-breaking is dominated by the untwisted moduli sector squark degeneracy is still expected.

5 Conclusions

String theory provides a window to study the unification of the gauge interactions with gravity. To bring this exploration to contact with experimental physics necessitates the construction of phenomenologically realistic models. The free fermionic models provide examples of such phenomenologically appealing models. Underlying these models there is a special $Z_2 \times Z_2$ orbifold. In this talk I discussed how the characteristic property of the $Z_2 \times Z_2$ orbifold, namely its cyclic permutation symmetry, may play a pivotal role in simultaneously understanding the fermion mass hierarchy and squark mass degeneracy.

Acknowledgments

It is a pleasure to thank Gerald Cleaver and Jogesh Pati for collaboration on the work reported in this talk. Work supported in part by DOE grant No. DE-FG-0586ER40272.

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